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ESTIMATION OF FAILURE
INITIATION IN LAMINATED
COMPOSITES BY MEANS
OF NONLINEAR SIX-FIELD
SHELL THEORY AND FEM

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LIST OF THE MOST IMPORTANT SYMBOLS AND NOTATIONS

$\underline{Q}(x)$	– tensor of mean rotary deformation of the shell cross sections
$T(x)$	– the structure tensor in the current configuration
$T_0(x)$	– the structure tensor in the reference (initial) configuration
x	– the position vector of initial configuration M
\mathbf{e}	– vector of shell strains
\mathbf{s}	– vector of shell stress and couple resultants
\mathbf{C}	– constitutive matrix
$\varphi^{(k)}$	– angle of fibre orientation in k^{th} lamina
$(a^{(k)}, b^{(k)}, c^{(k)})$	– orthogonal material coordinate system of k^{th} lamina
$\tilde{\mathbf{T}}_k$	– transformation matrix
$\mathbf{A}_{4 \times 4}$	– extensional stiffness matrix
$\mathbf{B}_{4 \times 4}$	– extension – bending coupling matrix
$\mathbf{D}_{4 \times 4}$	– bending stiffness matrix
$\mathbf{S}_{2 \times 2}$	– transverse shear stiffness matrix
$\mathbf{G}_{2 \times 2}$	– drilling resultants stiffness matrix
σ_{aa}	– normal stress in the 1 st material direction
σ_{bb}	– normal stress in the 2 nd material direction
σ_{ab}	– in-plane shear stress in the 2 nd material direction
σ_{ba}	– in-plane shear stress in the 1 st material direction
σ_a	– transverse shear stress on the a-c plane
σ_b	– transverse shear stress on the b-c plane
E_a	– elastic modulus of lamina in the 1 st material direction
E_b	– elastic modulus of lamina in the 2 nd material direction
G_{ab}	– in-plane shear modulus of lamina
G_{ac}	– transverse shear modulus on lamina a-c plane
G_{bc}	– transverse shear modulus on lamina b-c plane
ν_{ab}	– major Poisson's ratio of lamina
X_t	– absolute value of lamina tensile strength in the 1 st material direction
X_c	– absolute value of lamina compressive strength in the 1 st material direction
Y_t	– absolute value of lamina tensile strength in the 2 nd material direction
Y_c	– absolute value of lamina compressive strength in the 2 nd material direction
S_f	– lamina in-plane shear strength
S_t	– lamina transverse shear strength
σ_{nn}	– normal stress component acting on lamina failure plane
σ_{nl}	– shear stress component acting on lamina failure plane in the fibre direction
σ_{nt}	– shear stress component acting on lamina failure plane perpendicular to σ_{nl}
θ_f	– lamina failure plane inclination angle
η_{w1}	– Puck's criterion weakening factor