

ROMUALD SZYMKIEWICZ
SUILIANG HUANG
ADAM SZYMKIEWICZ

INTRODUCTION TO COMPUTATIONAL ENGINEERING HYDRAULICS

GDAŃSK 2016

GDAŃSK UNIVERSITY OF TECHNOLOGY PUBLISHING HOUSE
CHAIRMAN OF EDITORIAL BOARD
Janusz T. Cieśliński

REVIEWER
Michał Szydłowski

COVER DESIGN
Katarzyna Olszonowicz

Published under the permission
of the Rector of Gdańsk University of Technology

Gdańsk University of Technology publications may be purchased at
<http://www.pg.edu.pl/wydawnictwo/katalog>
orders should be sent to wydaw@pg.gda.pl

No part of this publication may be reproduced, transmitted, transcribed,
stored in a retrieval system or translated into any human or computer language
in any form by any means without permission in writing of the copyright holder.

© Copyright by Gdańsk University of Technology Publishing House, Gdańsk 2016

ISBN 978-83-7348-672-0

Gdańsk University of Technology Publishing House

Edition I. Publishing sheet 17,7, sheet printing 19,25, 1133/933

Printing and binding: Totem.com.pl, sp. z o.o., sp. k.
ul. Jacewska 89, 88-100 Inowrocław, phone 52 354 00 40

CONTENTS

PREFACE	7
---------------	---

Part 1. BASIC NUMERICAL TECHNIQUES APPLIED IN HYDRAULIC ENGINEERING

1. SOLUTION OF SYSTEMS OF LINEAR ALGEBRAIC EQUATIONS	11
1.1. Introduction to the problem	11
1.2. Direct methods	14
1.2.1. Systems with triangular matrices	14
1.2.2. Gauss elimination method	15
1.2.3. LU decomposition method	19
1.3. Iterative methods	25
1.3.1. Simple iterative methods	25
1.3.2. Representation of sparse matrices	28
1.3.3. Conjugate gradient method	29
1.3.4. Bi-conjugate gradient stabilized method	31
1.4. Ill-conditioned systems of equations	33
2. SOLUTION OF NONLINEAR ALGEBRAIC EQUATIONS AND THEIR SYSTEMS	35
2.1. Introduction to the problem	35
2.2. Methods for solving nonlinear algebraic equations	37
2.2.1. Bisection method	37
2.2.2. False position method	39
2.2.3. Newton method	41
2.2.4. Simple fixed point iteration	46
2.2.5. Steffenson method	50
2.2.6. Wegstein method	50
2.3. Methods for solving systems of nonlinear algebraic equations	51
2.3.1. Newton method	52
2.3.2. Picard method	54
3. CURVE FITTING USING LEAST SQUARES METHOD	56
4. SEARCHING EXTREME POINT FOR FUNCTION $f(x)$	60
4.1. Optimization problem as solution of nonlinear algebraic equation	61
4.2. Solution of optimization problem by dividing of interval containing the extreme point	61
4.2.1. Dividing on three equal parts	62
4.2.2. Dividing using golden number	64
5. SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS AND THEIR SYSTEMS	68
5.1. Introduction to numerical solution of ordinary differential equations	68

5.2. Numerical solution of the initial value problem for ordinary differential equation	69
5.3. Single step explicit methods	72
5.3.1. Euler explicit method	72
5.3.2. Improved Euler explicit method	73
5.3.3. Runge-Kutta method	74
5.4. Single step implicit methods	76
5.4.1. Euler implicit method	76
5.4.2. Implicit trapezoidal rule	77
5.4.3. Generalization of 1-step implicit formulas	78
5.4.4. Solution of nonlinear equations provided by implicit formulas	79
5.5. Solution of initial value problem for system of ordinary differential equations	79
5.6. Solution of boundary value problem for ordinary differential equations	82
6. INTRODUCTION TO SOLUTION PROBLEM OF PARTIAL DIFFERENTIAL EQUATIONS	86
6.1. Classification of 2 nd order PDE with 2 independent variables	86
6.2. Formulation of solution problem for PDEs	88
6.3. Numerical solution of PDEs	93
6.3.1. Solution of PDEs using FDM	94
6.3.2. Solution of PDEs using FEM	98
6.3.3. Properties of numerical methods for PDEs: convergence, consistency and stability	107

Part 2. HYDRAULIC ENGINEERING PROBLEMS SOLVED USING NUMERICAL TECHNIQUES

7. BASIC DEFINITIONS, RELATIONS AND EQUATIONS USED IN HYDRAULIC ENGINEERING	111
7.1. Discharge and average flow velocity	111
7.2. Bernoulli equation	112
7.3. Reynolds number	115
7.4. Steady flow in conduits	115
7.4.1. Steady flow in closed conduit	115
7.4.2. Steady uniform flow in open channel	120
8. ORIFICES, WEIRS AND SPILLWAYS	123
8.1. Discharge through orifices	123
8.2. Unsteady outflow of water from tank	125
8.3. Flow over weirs and spillways	129
8.4. Hydraulic jump and stilling basin	132
9. FLOW IN PIPES	134
9.1. Steady flow in pipes of water supply system	134
9.2. Unsteady flow in system: reservoir – conduit – chamber	139
10. STEADY OPEN CHANNEL FLOW	145
10.1. Normal depth in open channel	145
10.2. Critical, subcritical and supercritical flow	151
10.3. Designing of open channel cross section	155
10.4. Natural open channel	159

10.5. Rating curve $Q(h)$	160
10.6. Steady gradually varied flow in open channel	164
10.6.1. Governing equations for steady gradually varied flow	164
10.6.2. Solution of initial value problem for steady gradually varied flow equation	167
10.6.3. Solution of boundary value problem for steady gradually varied flow equations	172
11. UNSTEADY OPEN CHANNEL FLOW	180
11.1. Derivation of governing equations	180
11.1.1. Continuity equation	180
11.1.2. Dynamic equation	182
11.2. Simplified open channel flow equations	185
11.3. Storage equation	188
11.4. Numerical solution of advection equation	196
11.4.1. Classification of advection equation and required auxiliary conditions	196
11.4.2. Solution of linear advection equation using method of characteristics	198
11.4.3. Solution of linear advection equation using finite difference up-wind scheme ..	202
11.4.4. Accuracy analysis of numerical solution of linear advection equation using modified equation approach	204
11.5. Solution of Saint-Venant equations	206
11.6. Solution of nonlinear kinematic wave equation	211
12. GROUNDWATER FLOW	217
12.1. Introduction to the problem	217
12.2. Flow in saturated zone	219
12.3. Flow in unsaturated zone	226
12.4. Numerical solution of 1D groundwater flow equation using FDM	229
12.4.1. Solution using explicit scheme	231
12.4.2. Solution using Crank-Nicolson scheme	236
12.4.3. Solution of nonlinear equation using fully implicit scheme	240
12.5. Solution of 1D Richards equation for water flow in unsaturated soils	243
12.6. Numerical solution of 2D groundwater flow equation	254
12.6.1. Solution using explicit scheme	254
12.6.2. Solution using implicit scheme	256
12.6.3. Solution using alternating direction implicit method	258
12.7. Numerical solution of 2D groundwater steady flow equation	261
12.7.1. Solution of Poisson equation using FDM	261
12.7.2. Solution of steady groundwater flow in confined aquifer using FEM	271
13. POLLUTANTS TRANSPORT IN FLOWING WATER	280
13.1. Transport of substances dissolved in water	280
13.2. Heat transport by flowing water	283
13.3. Simplified forms of 1D transport equation	285
13.4. Numerical solution of 1D advection – diffusion transport equation	286
13.4.1. Solution using implicit scheme of FDM	287
13.4.2. Numerical diffusion in solution of advection-diffusion equation	291
13.4.3. Solution of advection-diffusion transport equation using modified FEM	294
13.5. Characteristics of processes represented in 1D transport equation	301
13.6. Advection-diffusion transport equation in environmental engineering	304
REFERENCES	306

PREFACE

As stated by Roberson, Cassidy and Chaudhry (1998): “Hydraulic engineering is the application of fluid mechanics and other science and engineering disciplines in the design of structures, and the development of projects and systems involving water resources”. An engineering consideration of any technical problem implies quantitative analysis, so obviously such an analysis in hydraulic engineering requires the computations of large range of the water flow problems. Indeed, the area of interest of hydraulic engineering covers all possible water flow cases, taking place in closed conduits (pipes and their systems), at the Earth’s surface (reservoirs, open channels) and in subsurface porous soils and rocks. The problems of water flow listed above are closely related to another discipline of science and technique, namely to environmental engineering, which focuses attention on the availability and quality of water resources. As the latter issue is strictly related to the contaminant migration in water bodies, mass transport phenomena should be considered as well.

Application of the basic principles of conservation to a control volume of water body leads to mathematical expression of the physical rules which govern the considered flow cases. These rules are written in the form of equations. Depending on the assumptions introduced during their derivation, they can take various particular forms. Consequently, a hydraulic engineer during his professional activity typically faces a very large spectrum of equations describing different cases of water flow. As the flow process of any fluid can be a very complex phenomenon, the corresponding equations are often relatively complicated and their solution can be a challenging task. From the fundamental course of fluid mechanics and hydraulics results that even simple flow cases are described by equations requiring a significant computational effort to be solved. This can be caused for instance by:

- nonlinearity appearing in many equations,
- non-uniform spatial distribution of the parameters,
- unsteadiness of flow process and the necessity of integration in long time intervals,
- irregular shape of integration domain,
- forced additional conditions which vary irregularly in time,
- large dimensions of the final systems of algebraic equations which must be solved.

The main aim of this book is to provide the reader with the fundamentals of computational methods used in hydraulic engineering. To make easier following of the presented problems by the reader, we decided to divide it into two parts.

In Part I some fundamental numerical techniques, the most frequently used in hydraulic and environmental engineering are briefly presented. Subsequent chapters of Part I are devoted to an overview of the typical computational techniques applied to solve of the systems of linear algebraic equations, the nonlinear equations and their systems and the ordinary differential equations frequently occurring in hydraulic engineering. Particular attention is focused on partial differential equations, playing very important role in flow

dynamics analysis, as well as on the numerical methods commonly applied for their solution, i.e. the finite difference method and the finite element method. Moreover some other related computational problems as the approximation and numerical solution of simple optimization problems will be presented.

Part 2 of this book deals with presentation and derivation of some governing equations typically encountered in hydraulic engineering. In the subsequent sections the equations for steady and unsteady flow in open channels, in closed conduits and in the subsurface including those for transport of matter dissolved in flowing water are considered. As our aim is not to give a complete course dealing with the hydromechanics and hydraulics, descriptions of the considered flow problems are presented as simple as possible. For more information the reader is referred to the sources presenting a more comprehensive description of the discussed problems. For this reason our considerations are limited rather to the one dimensional flow problems, with the exception of groundwater flow. Apart from short presentation of the considered problems and description of the methods applied for their solution, examples of computer codes written in Fortran language are also provided. Moreover, many computational examples illustrating considered problems and the applied approaches for their solutions are included into the text. However, we will consider problems related to the water flow and mass transport phenomena only. Computational problems dealing with the design of engineering structures are outside the scope of this book.

The presented book is dedicated primarily to the students of civil and environmental engineering. It is well known that the basic courses of the numerical techniques applied in hydraulic engineering and given at various technical universities are very similar. Noticing this fact the authors coming from China (College of Environmental Science and Engineering at Nankai University in Tianjin) and from Poland (Faculty of Civil and Environmental Engineering at Gdańsk University of Technology), decided to elaborate commonly a textbook covering the areas defined by the respective syllabuses at both Universities. During the preparation of this textbook the experiences resulting from our earlier co-operation on mathematical modeling of the transport and flow phenomena of the surface and ground waters appeared very useful. We hope that the presented textbook will be useful for the students of both Universities – in Tianjin and in Gdańsk. We hope as well as that this textbook can be interesting and helpful not only for our students, but also for other readers interested in the numerical methods and in hydraulic engineering.

We would like to acknowledge the support for our collaborative effort received from the following institutions: Ministry of Science and Technology, China (36–26), Ministry of Science and Higher Education, Poland, Rector of Gdańsk University of Technology, Dean of the Faculty of Civil and Environmental Engineering at GUT. Prof. Suiliang Huang is financially supported by NSFC (11672139) and NSFC of Tianjin (15JCYBJC22500).

The authors

Romuald Szymkiewicz

Suiliang Huang

Adam Szymkiewicz